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CYLINDRICAL TANGENT CONES AND THE SINGULAR SET OF MINIMAL SUBMANIFOLDS

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The question of what can be said about the structure of the singular set of minimal surfaces and the extrema of other geometric variational problems has remained largely open. Indeed, for minimal surfaces, apart from various upper bounds on the possible dimension of the singular set (see, e.g., [1], [5], [7], [8], [10], [13], [17], [19]), little has been known beyond the work of Jean Taylor [22], [23] and Brian White [24], [25] concerning $\bmod p$ and "(M, ε, δ)" minimizing hypersurfaces, where the tangent cones are of very special (and unvarying) type, and there are topological obstructions to perturbing away the singularities.

Here we prove rectifiability and local finiteness of measure of the singular set for various classes of minimal submanifolds, including for the first time cases where the tangent cones may have varying type and where there is no topological obstruction to perturbing away the singularities. For example we establish here (in Corollary 1 of §1) the $(n-2)$-rectifiability for the interior singular set of any $\bmod 2$ minimizing current of arbitrary codimension, and local finiteness for the $(n-2)$-dimensional measure of the "top-dimensional" part of this singular set. Perhaps more importantly, the work here produces some analytic machinery which seems to hold promise for further developments.

The key result of the present work is a technical decay lemma, Lemma 1 of §1. This lemma says roughly that, if $M \subset \mathbf{R}^{n+k}$ lies in a suitable "multiplicity one class" $\mathscr{M}$ of n-dimensional minimal submanifolds $\subset \mathbf{R}^{n+k}$ (described precisely in §1), and if M is close to $\mathbf{C}$ in a ball B_ρ in a suitable L^2 sense (made precise in Lemma 1), where $\mathbf{C} = \mathbf{C}_0 \times \mathbf{R}^m$ is a cylindrical cone having cross section $\mathbf{C}_0$ satisfying an "integrability condition" (see ‡‡ of §1), then either there is a significant "gap" in the part of the singular set consisting of points $X \in B_\rho$ where the density $\Theta_m(X)$ of M at $X \geq$ the density $\Theta_{\mathbf{C}}(0)$ of $\mathbf{C}$ at 0, or else there is a cylindrical

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