J. DIFFERENTIAL GEOMETRY
38 (1993) 501-517

WHITEHEAD GROUPS OF FINITE POLYHEDRA WITH NONPOSITIVE CURVATURE

BIZHONG HU

1. Introduction

Our results are on the Whitehead groups $\mathrm{Wh}\,\Gamma = K_1(Z\Gamma)/H_1\Gamma \times Z_2$ of some groups Γ relating to geometry. The strategy of using control topology plus geometry to study $\mathrm{Wh}\,\Gamma$ was previously used to prove $\mathrm{Wh}\,\pi_1 M = 0$ for closed flat manifolds M by Farrell and Hsiang in [8]. The following more general result was proved using ideas that involved sphere bundles and geodesic flows.

1.1. Theorem (F. T. Farrell and L. E. Jones [11] in first order). $\mathrm{Wh}\,\pi_1 M = 0$ *for any closed Riemannian manifold* M *with nonpositive curvature.*

In this paper we obtain a Whitehead group result concerning finite polyhedra of nonpositive curvature in two steps. The first step is to transform the problem to one about closed manifolds, by applying the idea of hyperbolization. In the second step we prove that the Whitehead group of a closed manifold with PL nonpositive curvature is zero as a result of improving some key ideas from [9], [11] and [15]. The meaning of a polyhedron with negative or nonpositive curvature was defined in [14] by Gromov to study hyperbolic groups. The result of this paper is as follows. It covers a major class of semihyperbolic groups.

1.2. Theorem. $\mathrm{Wh}\,\Gamma = 0$, $\Gamma = \pi_1 K$ *for any finite polyhedron* K *with nonpositive curvature.*

Note that this implies $K_1(Z\Gamma) = H_1\Gamma \times Z_2$, $\widetilde{K}_0(Z\Gamma) = 0$, $K_i(Z\Gamma) = 0$, $i \leq -1$. A previous result in this respect is the vanishing of Whitehead groups in the negative curvature case in [15]. One interest in extending that to the nonpositive curvature case is from the application of Theorem 1.2 to p-adic groups via their Euclidean buildings ([21]), which are the most interesting known examples of polyhedron nonpositive curvature structures. In particular there is

Received January 22, 1991 and, in revised form, July 14, 1992.