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PERESTROIKAS[1] OF OPTICAL WAVE FRONTS AND GRAPHLIKE LEGENDRIAN UNFOLDINGS

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0. Introduction

It is well known that parallels of given smooth hypersurfaces in $\mathbb{R}^n$ have Legendrian singularities [2], [3]. If we are concerned about the way in which these parallels change as we alter the distance, then we are concerned about perestroikas of wave front sets. This problem has been considered by several people [1], [2], [4], [5]. Roughly speaking, it has been shown that generic perestroikas of the singularities of parallels of smooth hypersurfaces are generic perestroikas of Legendrian singularities. In [15] Zakalyukin proved that generic perestroikas of Legendrian singularities are stable perestroikas of Legendrian singularities in the case $n \leq 5$ and classified these perestroikas. In this paper we shall consider the realization problem of the perestroikas of Legendrian singularities. In relation to this problem, Arnol′d [2, p. 40] mentioned that "one may find in the literature the statement that the local perestroikas of the wave fronts generated by the general Legendre mapping over space-time and of the equidistants (i.e. parallel) of the smooth hypersurface are the same. It seems this has never been correctly proved." However, he has corrected his mistakes in his other book [2, p. 60]: "Indeed, the non-trivial perestroikas A_1 and A_2 change the number of connected components of the Legendrian manifold. Hence they cannot occur as perestroikas of equidistant hypersurfaces." This fact was already known to Bill Bruce in 1983 [4]. We have the following natural question.

Question. What sort of a class of one-parameter families of Legendrian immersion germs is the correct class to describe perestroikas of parallels of hypersurfaces?

Here we shall give a candidate of this class which we call *graphlike Legendrian unfoldings*. Roughly speaking, a graphlike Legendrian unfolding is a Legendrian submanifold germ with a submersive generating function.

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[1]The author uses this word in honor of Professor V. I. Arnol′d.