J. DIFFERENTIAL GEOMETRY
38 (1993) 351-386

ON THE SMOOTH COMPACTIFICATION OF SIEGEL SPACES

WENXIANG WANG

Introduction

Let $X = \Gamma\backslash\Omega$ be a noncompact locally symmetric Hermitian space, where Ω is a bounded symmetric domain and Γ is an arithmetic subgroup acting on Ω. It is well known that X is quasiprojective[1], and the canonical Bergman metric on X induced from Ω is a Kähler-Einstein metric of negative curvature if X is smooth (it is the case where Γ is neat). Since the smooth compactifications of X were introduced in [1] from the toroidal embeddings, Mumford obtained the following results on X in his proof of noncompact Hirzebruch's proportionality [12]:

1. X is of logarithmic general type.
2. The Bergman metric g on X is a *good* singular Hermitian metric on any smooth toroidal compactification $\overline{X}$ of X. In other words, assuming that the boundary $D = \overline{X} - X$ is locally defined as $\prod_{i=1}^{k} z_i = 0$, then the volume form Φ of g behaves singularly along the boundary D as

$$(|z_1 \cdots z_k|^2 \Phi)^{-1} = O(\log^{2N} |z_1 \cdots z_k|)$$

for some integer $N > 0$.

To have broader and deeper applications of the theory on the locally symmetric Hermitian spaces in algebraic and differential geometry (see the references [15], [16] and [9]), people would like to understand more about X and its compactification $\overline{X}$ besides Mumford's work. One would like to completely understand the algebraic structures of the boundary divisor D and the canonical bundle $K_{\overline{X}}$ of $\overline{X}$ and to have a precise singular description of the canonical volume form Φ along D. The goal of this paper is to study these questions for the quotient of Siegel upper half spaces by an intensive investigation of their smooth toroidal compactifications.

Received February 24, 1992. This work is partially supported by the National Science Foundation grant DMS-8901303.

[1] A noncompact variety V is said to be quasiprojective if V is a Zariski open dense subset of a projective variety $\overline{V}$.