J. DIFFERENTIAL GEOMETRY
38 (1993) 315-349

VON NEUMANN INDEX THEOREMS FOR MANIFOLDS WITH BOUNDARY

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1. Introduction

The index theorem of Atiyah and Singer relates the index of elliptic operators on closed manifolds to characteristic numbers on the manifold. In the case of compact manifolds with boundary, Atiyah, Patodi, and Singer [3] showed that the index of a first-order elliptic operator of Dirac type is not the usual characteristic number; instead the difference is a term depending only on the boundary called the eta invariant.

In the case of noncompact manifolds without boundary the index of elliptic operators is not well defined since these operators in general are not Fredholm. In the case of elliptic differential operators, on infinite Galois coverings of closed manifolds, equivariant with respect to the Galois group, Atiyah (cf. [1]) introduced a real-valued index given by replacing the notion of dimension by a generalized dimension introduced by von Neumann. He proceeded to prove an index theorem for elliptic operators in the above context. In the case of noncompact manifolds arising as leaves of a foliation of a closed manifold, with holonomy invariant transverse measure, Connes (cf. [10]) proved a von Neumann index theorem for differential operators elliptic along the leaves of the foliation.

This then brings us to the question that is answered in this paper. What are the corresponding results in the case of noncompact manifolds with boundary, i.e., is there an analog of the Atiyah-Patodi-Singer index theorem for infinite Galois coverings of compact manifolds with boundary? Is there a similar index theorem in the case of foliations of compact manifolds with boundary, with the leaves intersecting the boundary transversally, and equipped with a holonomy invariant transverse measure? As in the usual Atiyah-Patodi-Singer index theorem one would like to know the analog of the usual eta invariant on the boundary. In the case of infinite Galois coverings these generalized eta invariants were introduced

Received June 6, 1989, and, in revised form, January 10, 1992.