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SPECTRAL DEGENERATION OF HYPERBOLIC RIEMANN SURFACES

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Abstract

Given a degenerating family S_l $(1 \geq 0)$ of Riemann surfaces with their canonical hyperbolic metrics, we work out in detail the spectral degeneration of the collars around the pinching geodesics in S_l. Using the spectral degeneration of the pinching collars, we show that the eigenvalues of S_l become dense at every point of the continuous spectrum $[\frac{1}{4}, +\infty)$ of S_0 and give upper and lower bounds for the rate of the clustering. Furthermore, we show that Eisenstein series, which are generalized eigenfunctions, of S_0 arise as limits of eigenfunctions of S_l as $l \to 0$.

1. Introduction

Let M_g $(g \geq 2)$ be the moduli space of compact Riemann surfaces of genus g, and $\overline{M_g}$ be the compatified moduli space of Riemann surfaces (see [11]). For any $S \in \overline{M_g}$, S has a canonical hyperbolic metric (of constant curvature -1), induced from the uniformization. From now on, we call such a surface with its canonical hyperbolic metric a hyperbolic Riemann surface.

With respect to this metric on S, we have the Beltrami-Laplace operator Δ_S, and its spectrum on $L^2(S)$ is denoted by spec(S). The spectrum is a very natural invariant of a manifold (see [19]). For generic $S \in M_g$, spec(S) uniquely determines S (see [38]).

It is therefore a natural question to consider the dependence of spec(S) on $S \in \overline{M_g}$. For $S \in M_g$, S is compact, and spec(S) is discrete. Furthermore, spec(S) changes real analytically in terms of suitable coordinates on the Teichmüller space, which is a covering space of M_g (see [37]).

On the other hand, for $S_0 \in \overline{M_g} \setminus M_g$, S_0 is complete, noncompact, and has finite area and cusps as its ends (see §2). Furthermore spec(S) = discrete part $\cup$ continuous spectrum $[\frac{1}{4}, +\infty)$ (see Proposition 2.5). The discrete part may be finite, and the continuous part has

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