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NONUNIFORM HYPERBOLIC LATTICES AND EXOTIC SMOOTH STRUCTURES

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0. Introduction

Let Θ_m denote the group of homotopy m-spheres where $m > 4$. Elements in Θ_m are equivalence classes of oriented manifolds homeomorphic to S^m. Two such manifolds Σ_1^m and Σ_2^m are equivalent provided there exists an orientation-preserving diffeomorphism between them. In this paper, $\mathbb{D}^{m+1}$ and S^m respectively denote the unit ball and unit sphere in $\mathbb{R}^{m+1}$; i.e.,

$$
\begin{aligned}
\mathbb{D}^{m+1} &= \{x \in \mathbb{R}^{m+1} \mid |x| \leq 1\}, \\
S^m = \partial\mathbb{D}^{m+1} &= \{x \in \mathbb{R}^{m+1} \mid |x| \leq 1\}.
\end{aligned}
\tag{0.01}
$$

Kervaire and Milnor proved in [13] that Θ_m is a finite abelian group.

Let M^m be a smooth m-dimensional manifold. A possible way to change its smooth structure, without changing its homeomorphism type, is to take its connected sum $M^m \# \Sigma^m$ with a homotopy sphere Σ^m. We showed in [9] that it is sometimes possible to change the smooth structure on a closed (real) hyperbolic manifold M^m in this way and still to have a negatively curved Riemannian metric on $M^m \# \Sigma^m$. But when M^m is noncompact (and connected), this method *never* changes the smooth structure on M^m. (See the proof of Corollary 1.5 for an argument verifying this statement.)

We use a different method in this paper, which can sometimes change the smooth structure on a noncompact manifold M^m. The method is to remove an embedded tube $S^1 \times \mathbb{D}^{m-1}$ from M^m and then reinsert it with a "twist". To be more precise, pick a smooth embedding $f: S^1 \times \mathbb{D}^{m-1} \to M^m$ and an orientation-preserving diffeomorphism $\varphi: S^{m-2} \to S^{m-2}$. Then a new smooth manifold $M_{f,\varphi}$ is obtained as a quotient space of the disjoint union

$$S^1 \times \mathbb{D}^{m-1} \amalg M^m - f(S^1 \times \operatorname{Int} \mathbb{D}^{m-1}), \tag{0.02}$$

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