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ISOPARAMETRIC SUBMANIFOLDS AND THEIR HOMOGENEOUS STRUCTURES

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0. Introduction

In [4] it was proved that given a submanifold of Euclidean space the representation on the normal space of the holonomy group of the normal connection is an s-representation (i.e., equivalent to the isotropy representation of a semisimple symmetric space). This result has important consequences for the isoparametric problem. For instance, it was used in [2] to give a geomeric characterization of the manifolds which are focal to the isoparametric submanifolds. The purpose of this article is to study in some detail how the homogeneity of isoparametric submanifolds is related to the precise knowledge of the normal holonomy groups of focal manifolds. As a consequence of this we will give a simple and geometric proof, without using Tits buildings, of the remarkable result of Thorbergsson.

Theorem (Thorbergsson [9]). *Every irreducible isoparametric submanifold of a Euclidean space of rank at least* 3 *is an orbit of an* s*-representation.*

Let M be an irreducible isoparametric submanifold of rank ≥ 3 of the Euclidean space. Then, due to the Homogeneous Slice Theorem (see [2]), there are abundantly many submanifolds of M which are homogeneous and isoparametric, namely, those obtained as the fibers of focalization. The problem is how to glue all this information together in order to conclude that M is homogeneous. We use, for this purpose, the techniques developed in [5] together with the results in [4]. Namely, we construct a canonical metric connection in TM such that the second fundamental form is parallel with respect to this connection and the usual connection in the normal bundle. The difference tensor between the Riemannian connection and the new one is also parallel. Then, by the main result in [5], M is an orbit. The method of constructing this connection is by gluing together the canonical connections of the submanifolds of M given by slices.

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