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# ON SIMPLY CONNECTED NONCOMPLEX 4-MANIFOLDS

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**Abstract**

We define a sequence $\{X_n\}_{n\geq 0}$ of homotopy equivalent smooth simply connected 4-manifolds, not diffeomorphic to a connected sum $M_1 \# M_2$ with $b_2^+(M_i) > 0$, $i = 1, 2$, for $n > 0$, and nondiffeomorphic for $n \neq m$. Each $X_n$ has the homotopy type of $7\mathbf{CP}^2 \# 37\overline{\mathbf{CP}}^2$. We deduce that for all but finitely many $n$ the connected sum of $X_n$ with a homotopy sphere is not diffeomorphic to a connected sum of complex surfaces, complex surfaces with reversed orientations and a homotopy sphere.

## 1. Introduction

A well-known conjecture, asserting that any smooth simply connected 4-manifold is diffeomorphic to a connected sum of complex surfaces and complex surfaces with reversed orientations ($S^4$ being the trivial connected sum), has recently turned out to be false, by work of Gompf and Mrówka [11]. Gompf and Mrówka produce infinite families of counterexamples via the following construction: in the smooth category it can be assumed that any elliptic fibration on a $K3$ surface has a cusp fiber $F$, and one can consider the *nucleus* $N$, a regular neighborhood of the union of $F$ with a smooth section of the fibration. A new surface can be obtained by performing "differentiable logarithmic transforms" on smooth fibers in the interior of $N$, and it is possible to find three disjoint nuclei in a $K3$ surface corresponding to three different elliptic structures, and do logarithmic transforms inside each of them. Gompf and Mrówka are able to compute, using previous work of Mrówka [13], the values of certain Donaldson invariants of the resulting manifolds. The classification of algebraic surfaces and the values of the invariants show that these manifolds are counterexamples to the conjecture.

The purpose of this paper, which draws part of its inspiration from the above ideas, is to produce new counterexamples to the same conjecture by a slightly different approach: let $X$ be the two-fold branched cover of

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