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# FLAT CONFORMAL STRUCTURES ON 3-MANIFOLDS, I: UNIFORMIZATION OF CLOSED SEIFERT MANIFOLDS

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**Abstract**

This is the first in a series of papers where we prove an existence theorem for flat conformal structures on finite-sheeted coverings over a wide class of Haken manifolds

## Introduction

A *flat conformal structure* on a manifold $M$ (of dimension $n > 2$) is a maximal atlas

$$K = \{(U_i, \varphi_i), \varphi_i: U_i \to V_i \subset \mathbb{S}^n, i \in I\}$$

with conformal transition maps $\varphi_i \circ \varphi_j^{-1}$. From more classical point of view a flat conformal structure (FCS) is a conformal class of conformally Euclidean Riemannian metrics on $M$. This definition is equivalent to the former one (see [34], [39], e.g.). The best-known way to construct FCS is by *uniformization*: If a Kleinian group $\Gamma$ acts freely and discontinuously on a domain $D \subset \mathbb{S}^n$, then a flat conformal structure $K_\Gamma$ naturally arises on the factor manifold $M = D/\Gamma$. For this structure $K_\Gamma$ the covering $p: D \to M$ is a conformal map. Such structures are called *uniformizable*, and $\Gamma$ is called the *uniformizing* group. Five 3-dimensional geometries [56] are conformally Euclidean: $\mathbb{S}^3$, $\mathbb{E}^3$, $\mathbb{H}^2 \times \mathbb{R}$, $\mathbb{S}^2 \times \mathbb{R}$, $\mathbb{H}^3$.

The abundance of FCS in the dimension 3 is provided by the following well-known result of Thurston.

**Theorem H** [51], [53], [57], [58], [59]. *Let $M$ be a closed atoroidal Haken 3-manifold. Then $M$ admits a hyperbolic structure.*

According to Kulkarni [38] FCS exists on connected sum of conformally flat manifolds. On the other hand, Goldman [9] has shown that any closed 3-manifold $M$, modeled on **Sol**- or **Nil**-geometry, does not admit a flat conformal structure.

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