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LINKING AND HOLOMORPHIC HULLS

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1. Introduction

If X and Y are disjoint compact oriented smooth submanifolds of a smooth oriented manifold M and are homologous to zero in M, then the linking number of X and Y, denoted $\operatorname{link}(X, Y)$ (or by $\operatorname{link}(X, Y; M)$ for clarity) is equal to the intersection number of V and Y, where (V, X) is a compact oriented submanifold with boundary in M. This can be taken as one of the several equivalent definitions of linking number; here the dimensions a, k, m of X, Y, and M respectively, satisfy $a+k = m-1$. We say that X and Y are linked if $\operatorname{link}(X, Y)$ is not zero. Our object is to apply this linking notion of Gauss to the geometry of holomorphic hulls. For example, in the case that the underlying manifold M is $\mathbf{C}^n$, our results say that the polynomially convex hull of one of the sets X or Y has a nonempty intersection with the other set, provided that X and Y are linked.

Now take M to be a Stein manifold and let X be a compact subset of M. Then the holomorphic hull of X is

$$\widehat{X} = \{p \in M: |f(p)| \leq \max\{|f(q)|: q \in X\} \text{ for all } f \in A(M)\}$$

where $A(M)$ is the space of all holomorphic functions on M. $\widehat{X}$ is a compact subset of M. In special cases arising from the maximum principle, $(\widehat{X}, X)$ is a smooth manifold with boundary which is foliated by complex manifolds with boundaries in X. In general however, $\widehat{X}$ is not so nice and may not contain any complex manifolds, or even continuous ones. Nevertheless the perception persists that the pair $(\widehat{X}, X)$ behaves like a manifold with boundary. This is the motivation for what follows. To adapt the above data on linking to this context we replace (V, X) with $(\widehat{X}, X)$ where now X is an arbitrary compact subset of M. As before Y is an oriented manifold disjoint from X and homologous to zero in M. Then, when X and Y are linked in an appropriate sense, the previous consequence that V and Y have a nonzero intersection number will be

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