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ON THE GENERALIZED CYCLE MAP

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Abstract

We relate Friedlander-Mazur cycle map [14] for projective varieties with Almgren's isomorphism [1] for integral currents. As a consequence we obtain the naturality of the F-M map, extend it to quasiprojective varieties, show its compatibility with localization sequences and pull-backs, and use it to compute several examples. As a corollary of our main result, we give a characterization of those varieties for which the cycle map is an isomorphism, as the ones whose space of p-dimensional algebraic cycles is weakly homotopy equivalent to the space of $2p$-dimensional topological cycles, for all p.

1. Introduction

The aim of this paper is to study and extend, under the light of geometric measure theory, some properties of the "Lawson homology" of quasiprojective varieties.

The Lawson homology $L_pH_n(X)$ of a closed complex projective variety $X \subseteq \mathbb{P}^N$ was first defined by E. Friedlander in [11], who was building on the fundamental work of H. B. Lawson [20]. The definition was subsequently extended to include quasiprojective varieties in [24]. For a closed projective variety X, the Lawson homology group $L_pH_n(X)$, $n \geq 2p$, was originally defined as the homotopy group $\pi_{n-2p}(\mathscr{C}_p(X)^+)$, where $\mathscr{C}_p(X)$ is the Chow monoid of effective p-cycles supported in X and $\mathscr{C}_p(X)^+$ is a "homotopy group completion" of $\mathscr{C}_p(X)$. Lawson homology is a *covariant* functor from the category of quasiprojective varieties and proper morphisms to the category of abelian groups, and a *contravariant* functor from the category of quasiprojective varieties and flat maps to the category of abelian groups. In this paper we only consider the Lawson homology of varieties over the complex numbers and its behavior under proper maps. In particular, when we assert that Lawson homology is functorial, we mean

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