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INSTANTONS ON $n\mathbb{CP}_2$

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0. Introduction

On a complex surface equipped with an Hermitian metric the splitting of the 2-forms into self-dual and anti-self-dual components is compatible with the splitting into forms of different types induced by the complex structure: $\Lambda^2_+ \otimes \mathbf{C} = \Lambda^{0,2} \oplus \Lambda^{2,0} \oplus \omega\Lambda^{0,0}$, and $\Lambda^2_- \otimes \mathbf{C} = \ker \omega\wedge : \Lambda^{1,1} \to \Lambda^{2,2}$, where ω is the positive $(1, 1)$-form defined by the metric and the complex structure. Thus a connection with anti-self-dual curvature on a unitary bundle over such a surface automatically acquires a compatible holomorphic structure by the Newlander-Nirenberg theorem. It is this key fact which underlies Donaldson's result [12] showing the equivalence of moduli of anti-self-dual connections and stable holomorphic bundles on an algebraic surface, a result of central importance in the evolving gauge-theoretic study of smooth 4-manifolds.

It is perhaps less well-known that the same fact can be used to describe moduli of *self*-dual Yang-Mills connections ("instantons") on oriented 4-manifolds without complex structures: let $\widetilde{\mathbb{C}}^2$ denote a modification of the complex plane consisting of n blow-ups and let ω be a positive $(1, 1)$-form on this space. An ω-anti-self-dual solution of the Yang-Mills equations is then a holomorphic bundle with hermitian connection whose curvature F satisfies $\omega \wedge F = 0$. If the solution has finite L^2 action and ω is suitable asymptotically flat, the bundle and connection extend to the one-point compactification by Uhlenbeck's theorem [30]. Since this one-point compactification is diffeomorphic to a connected sum of n copies of the reverse-oriented complex projective plane, flipping the orientation yields a self-dual solution of the Yang-Mills equations on this last space, that is, an instanton on $n\mathbb{CP}_2$.

There is a smooth orientation-reversing map $\overline{\pi} : \widetilde{\mathbb{P}}_2 \to n\mathbb{CP}_2$ collapsing the line L_∞ at infinity to a point y_∞ (an "antiholomorphic blow-down"). Under this map the instanton on $n\mathbb{CP}_2$ pulls back to an extension of the

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