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ISOSPECTRAL CLOSED RIEMANNIAN MANIFOLDS WHICH ARE NOT LOCALLY ISOMETRIC

CAROLYN GORDON

Two compact Riemannian manifolds are said to be *isospectral* if the associated Laplace-Beltrami operators have the same eigenvalue spectrum. Milnor [18] constructed the first pair of isospectral, nonisometric manifolds, a pair of 16-dimensional flat tori. Many new examples and also techniques for constructing examples have appeared in the past decade; see for example [2], [3], [4], [6], [10], [11], [12], [13], [22] and [25] or the surveys [1], [2], [5] and [8]. However, in all these examples, the isospectral manifolds are locally isometric; in particular, in all the examples of isospectral closed Riemannian manifolds, the manifolds have a common Riemannian covering.

Recently, Zoltan Szabo [24] constructed the first examples of isospectral Riemannian manifolds (with boundary) which are not locally isometric. The manifolds are geodesic balls in different harmonic manifolds of nonpositive curvature. These manifolds are first introduced in [23].

The purpose of this article is to construct pairs of isospectral *closed* Riemannian manifolds with no common covering. The manifolds involved are two-step nilmanifolds of Heisenberg type. In each case, the construction gives two continuous families F_1 and F_2 of Riemannian manifolds all of which are isospectral. Those in a given family are locally isometric but not isometric. The manifolds in F_1 are not locally isometric to those in F_2. (See Remark 2.2.)

After describing the general construction, we will give specific examples and describe the geometry of two of the pairs of isospectral but not locally isometric manifolds. We will note differences both in the size of the isometry groups of the simply connected covers and in the curvature.

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