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MORSE INEQUALITIES FOR PSEUDOGROUPS OF LOCAL ISOMETRIES

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Abstract

For complete pseudogroups of local isometries with compact space of orbits, the method of Witten is used to prove Morse inequalities for the invariant cohomology. An inequality is also proved for the cohomology of the space of orbit closures. These results are applied to the basic cohomology of Riemannian foliations, relating the tautness character to basic functions with no degenerate critical leaf closures.

Introduction

Let $\mathscr{H}$ be a complete pseudogroup of local isometries of a Riemannian manifold M such that the space of $\mathscr{H}$-orbits, $M/\mathscr{H}$, is compact. In this paper we prove Morse inequalities for the invariant cohomology $H(M)_{\mathscr{H}}$ (the cohomology of the complex $(A(M)_{\mathscr{H}}, d)$ of invariant differential forms).

Definition. An $\mathscr{H}$-orbit closure F is called a critical orbit closure of a function $f \in C^{\infty}(M)_{\mathscr{H}}$ if F contains critical points of f. F is called a nondegenerate critical orbit closure if F is the disjoint union of nondegenerate critical submanifolds. In this case, the index of F is well defined as the index of any of its connected components, and denoted by $m_F(f)$ (or simply m_F). The function f is called a nondegenerate $\mathscr{H}$-Morse function if all of its critical orbit closures are nondegenerate. For such a function, let $\mathrm{Crit}_{\mathscr{H}}(f)$ be the set of its critical orbit closures.

(See e.g. [4] or [6] for the degenerate Morse theory that will be used in this paper.)

If f is a nondegenerate $\mathscr{H}$-Morse function, then $\mathrm{Crit}_{\mathscr{H}}(f)$ is a discrete subset of the space of $\mathscr{H}$-orbit closures, $M/\overline{\mathscr{H}}$. Thus $\mathrm{Crit}_{\mathscr{H}}(f)$ is finite because $M/\mathscr{H}$ is compact. The existence of nondegenerate $\mathscr{H}$-Morse functions follows easily from the case solved by A. G. Wasserman [24], where $\mathscr{H}$ is generated by an action of a compact Lie group.

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