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ON MINIMAL HYPERSURFACES WITH CONSTANT SCALAR CURVATURES IN S^4

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Introduction

Let M^n be a piece of minimally immersed hypersurface in the unit sphere S^{n+1}, and h its second fundamental form. Denote by R and S its scalar curvature and the square norm of h, respectively. It is well known that $S = n(n-1) - R$ from the structure equations of both M^n and S^{n+1}. In 1968, J. Simons [9] observed that if $S \leq n$ everywhere and either M^n is compact or S is constant, then $S \in \{0, n\}$. Clearly, M^n is contained in an equatorial sphere if $S = 0$. And when $S = n$, M^n is indeed a piece of a product of spheres, due to the works of Chern, do Carmo, and Kobayashi [4] and Lawson [6]. These two kinds of hypersurfaces are the so-called isoparametric ones of types 1 and 2, respectively.

Definition. A hypersurface of S^{n+1} is called *isoparametric of type* g if it has g distinct constant principal curvatures of constant multiplicities.

The classification of isoparametric hypersurfaces in spheres is far from being completed although the study has been very fruitful. An interested reader is referred to the book of Cecil and Ryan [2]. Here we will only mention a pioneering work of E. Cartan [1] and leave our pursuit on this topic in [3].

Theorem [Cartan, 1939]. *There exist minimal isoparametric hypersurfaces of type* 3 *in spheres only in the dimension of* 3, 6, 12, *and* 24. *Moreover, it is unique in each of such dimensions up to a rotation on the sphere.*

These hypersurfaces will be referred to as *Cartan's minimal hypersurfaces.*

We are concerned about the following conjecture posed by Chern [11].

Chern Conjecture. *For any* $n \geq 3$, *the set* R_n *of all the real numbers each of which can be realized as the constant scalar curvature of a closed minimally immersed hypersurface in* S^{n+1} *is discrete.*

There have been many works in this regard (e.g. [5], [7], [8], [10]). In the special case of $n = 3$, Peng and Terng [7], [8] derived the following

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