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ON THE MODULI SPACE OF VECTOR BUNDLES ON THE FIBERS OF THE UNIVERSAL CURVE

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Dedicated to the memory of S. K. Pichorides

Abstract

In this paper we describe the Picard group of the variety $\mathscr{U}(r, d)$ which parametrizes semistable vector bundles of rank r and degree d on the fibers of the universal curve $\mathscr{C}_g$. The bundle $\mathscr{U}(r, d)$ lies over the moduli space $\mathscr{M}_g^0$ of smooth curves of genus g $(g \geq 3)$ without automorphisms.

1. Introduction

We denote by $\mathscr{M}_g^0$ the moduli space of smooth curves of genus g $(g \geq 3)$ without automorphisms. To this space we can associate various varieties: The universal curve $\pi: \mathscr{C}_g \to \mathscr{M}_g^0$ which is a bundle with fiber the curve C over the point $[C] \in \mathscr{M}_g^0$; the variety $q: \mathscr{U}(r, d) \to \mathscr{M}_g^0$ with fiber over $[C]$ the space $U_C(r, d)$, which parametrizes semistable vector bundles of rank r and degree d on C—for the definition see [9]. In the special case when $r = 1$, this becomes the Jacobian variety $p: \mathscr{J}^d \to \mathscr{M}_g^0$ of degree d with fiber $J^d(C)$ over the point $[C]$, which parametrizes line bundles of degree d on C.

The Picard groups of $\mathscr{M}_g^0$ and $\mathscr{C}_g$ have been described by Harer, Arbarello and Cornalba (see [1]). The $\operatorname{Pic}\mathscr{M}_g^0$ is generated by the determinant λ of the Hodge bundle. On the other hand, the restriction of a line bundle on $\mathscr{C}_g$ to the fibers of π is something "canonical", namely a multiple of the canonical bundle (Franchetta's problem, see [1]). Therefore the relative Picard group $\operatorname{Pic}(\mathscr{C}_g/\mathscr{M}_g^0)$ is generated by the relative dualizing sheaf ω_π of the family π and the $\operatorname{Pic}\mathscr{C}_g$ is the free abelian group with generators ω_π and $\pi^*\lambda$.

In this paper we prove that a similar phenomenon holds for line bundles on $\mathscr{U}(r, d)$. The restriction of a line bundle on $\mathscr{U}(r, d)$ to a fiber

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