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INDEX THEORY FOR CERTAIN COMPLETE KÄHLER MANIFOLDS

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1. Introduction and notation

Let $\overline{M}$ be a compact Kähler manifold of real dimension n with Kähler form ω', and let $\mathscr{D} = \mathscr{D}_1 \cup \cdots \cup \mathscr{D}_N \subset \overline{M}$ be a divisor with simple normal crossings. The noncompact manifold $M = \overline{M} - \mathscr{D}$ may be endowed with a complete finite volume metric h with Poincaré growth at the $\mathscr{D}_i$ (see for example [2]) determined by the Kähler form

$$\omega = T\omega' - \sum_{j=1}^{N} \partial\overline{\partial} \log\log^2 |\sigma_j|^2. \tag{1}$$

Here $|\cdot|$ denotes a Hermitian norm on the line bundle $[\mathscr{D}_j]$, σ_j is a section of $[\mathscr{D}_j]$ defining $\mathscr{D}_j$, and T is a large real constant. We normalize the Kähler form so that the Kähler form on $\mathbf{C}$ corresponding to the usual metric $dx^2 \oplus dy^2$ is $\frac{1}{2} dz \wedge d\overline{z} = -i\, dx \wedge dy$. Thus the metric determined by a Kähler form ω is given by $(v_1, v_2) = i\omega(v_1, Jv_2)$, where J is the complex structure operator. For a multi-index I, set

$$\mathscr{D}_I = \bigcap_{i \in I} \mathscr{D}_i. \tag{2}$$

The manifold $\mathscr{D}_I' \equiv \mathscr{D}_I - \bigcup_{J \supset I,\ J \neq I} \mathscr{D}_J$ inherits a complete metric h_I determined by $\omega|_{D_I}$.

Let E be a unitary flat bundle over M, and F a hermitian holomorphic bundle over M. Denote by $H_2^{\cdot}(M, h, E)$ the L^2 cohomology of (M, h) with coefficients in E. The cup product pairing defines a quadratic form Q on $H_2^{n/2}(M, h, E)$. When this group is finite-dimensional, we call the signature of Q the L^2-signature of (M, h, E). We define the L^2-Euler characteristic of (M, h) to be $\sum_p (-1)^p \dim H_2^p(M, h, \mathbf{C})$, when each of these groups is finite dimensional. Similarly, given a hermitian

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