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EQUIVALENCE CLASSES OF POLARIZATIONS AND MODULI SPACES OF SHEAVES

ZHENBO QIN

Introduction

Let X be a smooth algebraic variety over the complex number field $\mathbf{C}$ with dimension n larger than one. For fixed c_1 in $\mathrm{Pic}(X)$, c_2 in $A^2_{\mathrm{num}}(X)$ which is the Chow group of codimension-two cycles on X modulo numerical equivalence and a polarization L on X, let $\mathcal{M}_L(c_1, c_2)$ be the moduli space of locally free rank-two sheaves stable with respect to L in the sense of Mumford-Takemoto such that their first and second Chern classes are c_1 and c_2 respectively. In this paper, we consider the problem: *what is the difference between* $\mathcal{M}_{L_1}(c_1, c_2)$ *and* $\mathcal{M}_{L_2}(c_1, c_2)$ *where* L_1 *and* L_2 *are two different polarizations?*

The understanding of this problem has two important implications. The first is in algebraic geometry. If one knows the structure of some moduli space $\mathcal{M}_L(c_1, c_2)$, then one will know the structure of any other moduli space $\mathcal{M}_{L'}(c_1, c_2)$ by comparing it with $\mathcal{M}_L(c_1, c_2)$. The author has applied this idea to the case where X is a ruled surface (for instance, see [21]); the results will appear elsewhere. The second implication is in gauge theory where X is an algebraic surface. When the geometric genus p_g of X is positive, the polynomials defined by Donaldson [6] are differential invariants. When p_g is zero, via the results in [4], these polynomials are defined on chambers of certain type (c_1, c_2); from the work of Mong [18] and Kotschick [15], one sees that we need to understand the difference between moduli spaces in order to compute these polynomials.

Our approach to the problem is to develop a theory about equivalence classes, walls and chambers of type (c_1, c_2) for polarizations on X. This is done in Chapter I. Fix c_1 and c_2 as before. Let L_1 and L_2 be two polarizations on X. We say that L_1 and L_2 are *equivalent* if every locally free rank-two sheaf V with first and second Chern classes c_1 and c_2, respectively, is L_1-stable if and only if it is L_2-stable.

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