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THE HEAT TRACE ON SINGULAR ALGEBRAIC THREEFOLDS

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1. Introduction

Let X be a complex projective algebraic threefold with isolated singularity set Σ. Consider the Laplacian $\overline{\Delta} = \overline{\delta d}$ with respect to the induced Fubini-Study metric on the noncompact smooth locus $X - \Sigma$ acting on square integrable functions. In [7], we showed that $\overline{\delta} = \overline{d}_0^* = \overline{d}^*$, which implied the selfadjointness of the Laplacian $\overline{\Delta}$. The main result of this paper is

1.1. **Theorem.** *The trace of the heat operator* $\overline{e}^{t\overline{\Delta}}$ *is finite and satisfies*

$$\mathrm{Tr}\, e^{-t\overline{\Delta}} \leq K t^{-3}$$

for $t \in (0, T]$, *suitable* $T > 0$, *and* $K > 0$.

1.2. **Remarks.** The corresponding facts for curves and surfaces are respectively due to Cheeger [2], [3] and Nagase [6].

2. Reduction to local problems

Let X, Σ be as above. Then by the main results of §2,3 of [7], we may decompose

$$X - \Sigma = M \cup \left(\bigcup_{\alpha=1}^{m} W_\alpha^b \right), \tag{1}$$

where $M = \{x \in X - \Sigma : d(x, \Sigma) \geq b\}$ for some fixed $b \in (0, 1)$, and the W_α^b are sets of the type W_{I}^b, W_{II}^b, W_{III}^b, which were introduced in [7, §2, 3]. Similarly, the ε-truncation X_ε of X is defined as

$$X_\varepsilon = \{x \in X - \Sigma : d(x, \Sigma) \geq \varepsilon\} = M \cup \left(\bigcup_{\alpha=1}^{m} W_\alpha^b(\varepsilon) \right), \tag{2}$$

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