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CONVERGENCE OF CURVATURES IN SECANT APPROXIMATIONS

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1. Introduction

It has long been known that a closed polyhedron P in Euclidean space $\mathbf{E}^n$ admits certain *curvature measures* analogous to classical curvature integrals (cf. [3], [11], [17], [1], [15]). If $P^1, P^2, \ldots$ is a sequence of such polyhedra converging to a smooth submanifold of $M \subset \mathbb{E}^n$, it is natural to ask whether the curvature measures of the P^i converge to the corresponding curvature integrals of M. (In view of well-known examples in area theory (cf. [16, I.1.10]) it is of course necessary to take some care in formulating the hypothesis precisely.) An intrinsic analogue of this equation has been answered positively in [5] by Cheeger, Müller and Schrader, who have also asserted that their method applies equally well to the extrinsic question above. Our aim in the present article is to give a solution to the extrinsic problem that is conceptually much simpler than the solution of [5].

Our approach rests on the observation that the curvature measures (or integrals) of polyhedra P (or smooth submanifolds M) in $\mathbb{E}^n$ may be computed in a universal way from a certain *integral current*, canonically associated to P (or M), living in the tangent sphere bundle $S\mathbb{E}^n \cong \mathbb{E}^n \times S^{n-1}$ (cf. [19], [20], [6]). If M is smooth, then this current is given by integration over the canonically oriented $(n-1)$-manifold $N(M)$ of unit normals to M. We may associate a similar object $N(P)$ to a polyhedron P; although $N(P)$ is no longer a submanifold of $S\mathbb{E}^n$, it is an integral current of dimension $n-1$, called the *normal cycle* to P. To obtain the curvature measures of P, we observe that there are universal differential $(n-1)$-forms $\kappa_0, \cdots, \kappa_{n-1}$ in $S\mathbb{E}^n$ such that the curvature measures of P are given by

$$\Phi_i^P = \pi_\#(N(P) \llcorner \kappa_i), \qquad i = 0, \cdots, n-1,$$

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