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DEFORMATIONS OF FLAT CONFORMAL STRUCTURES ON A HYPERBOLIC 3-MANIFOLD

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Abstract

We show that a particular closed hyperbolic 3-manifold with a totally geodesic hypersurface of genus two admits a real two-dimensional family of flat conformal deformations that are distinct from the deformations obtained by bending along the totally geodesic hypersurface. The construction is quite general and can be applied to other not necessarily hyperbolic manifolds; it follows from a more general theory of bending hyperbolic cone manifolds along totally geodesic hyperplanes intersecting at the singular set.

1. Introduction

It is well known by Mostow's rigidity theorem [16] that the hyperbolic structure on a closed hyperbolic manifold M of dimension $n \geq 3$ is rigid. On the other hand, in the category of flat conformal structures there may be nontrivial deformations. (Recall that a flat conformal structure on M is a maximal atlas of charts modelled on subsets of S^n such that, locally, the transition functions are restrictions of the group of conformal automorphisms of S^n.) The most obvious deformations are those that correspond to "bending" along complete, totally geodesic hypersurfaces; these have been studied by several authors (see for example [2], [10], [11], [13] and [14]). At the same time, Apanasov has constructed a different type of deformations called "stamping deformations" and studied the general problem of the deformation space of flat conformal structures in a series of papers (see [1]–[6]).

In this paper, we construct a different type of deformation for simply connected conformally flat three-manifolds. The problem with applying this to obtain deformations of closed hyperbolic three-manifolds is that we need to be able to apply our deformations in an equivariant manner to the universal cover of the three-manifolds. We show that this is possible for a particular closed three-manifold X, thus obtaining a real two-dimensional

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