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RIGIDITY OF PROPER HOLOMORPHIC MAPS BETWEEN SYMMETRIC DOMAINS

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In 1973 Mostow [21] proved a strong rigidity theorem to the effect that for compact Riemannian locally symmetric spaces of negative Ricci curvature the fundamental groups essentially determine the geometry (with obvious exceptions). Four years later, Margulis's Superrigidity Theorem gave as a consequence that for irreducible Riemannian locally symmetric spaces X and Y of negative Ricci curvature and finite volume, any "nondegenerate" continuous map $f: X \to Y$ is homotopic to an isometric immersion (up to normalizing constants), provided that X is of rank ≥ 2. In the last decade much work has been done in connection with the above rigidity theorems of Mostow and Margulis, e.g., [3], [4], [5], [6], [10], [19], etc. In 1978 Siu studied the strong rigidity of Kähler structures of compact quotients of bounded symmetric domains and obtained

Theorem 1 [24], [25]. *Let M be a compact quotient of an irreducible bounded symmetric domain of complex dimension ≥ 2. Suppose that X is a compact Kähler manifold homotopic to M. Then X is either biholomorphic or conjugate-biholomorphic to M.*

Siu's theorem covers the Hermitian case of Mostow's strong rigidity theorem for the reasons that any two compact $K(\pi, 1)$-spaces with isomorphic 160fundamental groups are homotopic and that any biholomorphism between Hermitian (locally) symmetric spaces of noncompact type is necessarily an isometry (up to normalizing constants). In connection with this, it is natural to study the rigidity problem for holomorphic mappings between Hermitian locally symmetric spaces of noncompact type. Indeed the following theorem was established by Mok in the compact case and by Mok and To in the finite-volume case as a consequence of their metric rigidity theorems.

Theorem 2 [16], [26]. *Let (X, g) be a Hermitian locally symmetric space of finite volume uniformized by an irreducible bounded symmetric domain Ω of rank ≥ 2. Suppose (Y, h) is any Hermitian locally*

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