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METRIC PROPERTIES OF MANIFOLDS BIMEROMORPHIC TO COMPACT KÄHLER SPACES

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Introduction

A goal of this paper is to prove that: "Every compact complex manifold M bimeromorphic to a compact Kähler manifold M' is balanced; that is, M has a hermitian metric with Kähler form ω such that $d\omega^{N-1} = 0$, $N = \dim M$" (Corollary 4.5). Of course, every Kähler manifold is balanced; the interest of the above result stems from the fact that we find out a metric property which transfers from M' to M, while it is well known that the Kähler property is not stable under bimeromorphic maps.

This introduction is mainly devoted to outline the background.

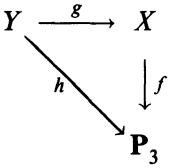
Let M and $\widetilde{M}$ be compact complex manifolds and $f: \widetilde{M} \to M$ be a modification. It is well known that:

(1) If f is a blow-up of M with smooth center and M is Kähler, then $\widetilde{M}$ is Kähler too [4],

however

(2) in general, if f is a modification and M is Kähler, $\widetilde{M}$ fails to be Kähler.

A counterexample is given in [12, p. 505] by a compact non-Kähler threefold X and a modification $f: X \to \mathbf{P}_3$. In order to illustrate Chow's lemma, Hironaka builds up also a projective threefold Y and a commutative diagram



where g and h are obtained as a finite sequence of blow-ups with smooth centers.

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