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RATIONAL CONNECTEDNESS AND BOUNDEDNESS OF FANO MANIFOLDS

JÁNOS KOLLÁR, YOICHI MIYAOKA & SHIGEFUMI MORI

0. Introduction

Fano manifolds are, by definition, smooth projective varieties with ample first Chern class (= anticanonical class). They are of special interest from the viewpoint of classification theory via minimal models; in fact, a principal goal of the minimal model program is to decompose a general algebraic variety into the Fano-like part and the minimal part (cf. [5], [11]).

Two-dimensional Fano manifolds are usually called Del Pezzo surfaces. Their classification into ten families of rational surfaces is an immediate consequence of Castelnuovo's criteria for rationality and minimality and Enriques' theory of adjunction. However, the systematic study of Fano manifolds initiated by G. Fano has revealed that their structure is not so simple in higher dimensions. For instance, the list of Fano 3-folds consists of 104 deformation classes, many of which are not rational [4], [12]. In dimension ≥ 4, their complete classification is thus virtually impossible and we should rather be concerned with vague but more accessible questions:

Question 1. Does the set of n-dimensional Fano manifolds form a bounded family?

Question 2. What can be said about geometric properties shared by the Fano manifolds in common?

The aim of this paper is to answer these questions: rational connectedness and boundedness.

A variety X is said to be *rationally connected* if two general points can be joined by an irreducible rational curve on X. Rational connectedness is a birational and deformation invariant, thus fitting well into the classification theory [8]. Roughly speaking, it is a crude generalization of unirationality, which is often too subtle to deal with in a general framework. Through crude, this generalization is natural enough to yield most of the geometric properties known for unirational varieties such as

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