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THE INTEGRAL OF THE SCALAR CURVATURE OF COMPLETE MANIFOLDS WITHOUT CONJUGATE POINTS

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Abstract

We prove that the integral of the scalar curvature of a complete manifold M without conjugate points is nonpositive and vanishes only if M is flat, provided that the Ricci curvature on the unit tangent bundle SM has an integrable positive or negative part.

Introduction

A complete Riemannian manifold M is said to be without conjugate points if the geodesics of M contain no pair of conjugate points, equivalently, if any two distinct points of its universal covering, endowed with the induced metric, are joined by a unique geodesic. If the sectional curvature of M is nonpositive, then M has no conjugate points. However, there exist compact and complete noncompact manifolds without conjugate points and with sectional curvature of both signs (see [2] or [7] for examples).

The object of this paper is to prove the following result.

Theorem A. *Let M be a complete manifold without conjugate points. Suppose that the Ricci curvature on the unit tangent bundle SM has an integrable positive or negative part. Then the integral of the scalar curvature of M is nonpositive and vanishes only if M is flat.*

Theorem A generalizes results of several authors. The inequality is due to Cohn-Vossen [4] when M is two-dimensional and simply connected. The result was obtained by E. Hopf [8] for surfaces with finite volume and Gaussian curvature bounded from below. In [6] Green extended the result of E. Hopf for complete n-dimensional manifolds with finite volume and sectional curvature bounded from below. Finally, in [9] Innami proved the theorem for complete n-dimensional manifolds with the additional hypotheses that the integral of the Ricci curvature is finite, and the nonwandering set of SM decomposes into at most countably many invariant

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