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ON THE LAPLACIAN AND THE GEOMETRY OF HYPERBOLIC 3-MANIFOLDS

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Abstract

Let $N = \mathbf{H}^3/\Gamma$ be an infinite volume hyperbolic 3-manifold which is homeomorphic to the interior of a compact manifold. Let $\lambda_0(N) = \inf \operatorname{spec}(-\Delta)$ where Δ is the Laplacian acting on functions on N. We prove that if N is not geometrically finite, then $\lambda_0(N) = 0$, and if N is geometrically finite we produce an upper bound for $\lambda_0(N)$ in terms of the volume of the convex core. As a consequence we see that $\lambda_0(N) = 0$ if and only if N is not geometrically finite. We also show that if N has a lower bound for its injectivity radius and is not geometrically finite, then its limit set L_Γ has Hausdorff dimension 2.

1. Introduction

In this paper we will study the relationship between the geometry of infinite volume hyperbolic 3-manifolds and the bottom λ_0 of the spectrum of the Laplacian. We will also study the relationship between spectral information and the measure-theoretic properties of the limit set. These relationships have been studied extensively by Patterson (cf. [28], [27]) and Sullivan (cf. [32], [33]), and much of this paper may be regarded as an extension of their work. Recall that a hyperbolic 3-manifold is said to be *topologically tame* if it is homeomorphic to the interior of a compact 3-manifold. Our first result is:

Theorem A. *Let N be an infinite volume, topologically tame hyperbolic 3-manifold. Then $\lambda_0(N) = 0$ if N is not geometrically finite. Moreover, there exists a constant K such that if N is geometrically finite, then*

$$\lambda_0(N) \leq K \frac{|\chi(\partial C(N))|}{\operatorname{vol}(C(N))},$$

where $\operatorname{vol}(C(N))$ denotes the volume of N's convex core.

Combining Theorem A with work of Lax and Phillips ([20], [21]) we show that λ_0 detects whether or not a topologically tame hyperbolic 3-manifold is geometrically finite.

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