J. DIFFERENTIAL GEOMETRY
36 (1992) 331-348

PUKANSZKY'S CONDITION AND SYMPLECTIC INDUCTION

C. DUVAL, J. ELHADAD & G. M. TUYNMAN

Abstract

Pukanszky's condition is a condition used in obtaining representations from coadjoint orbits. In order to obtain more geometric insight into this condition, we relate it to symplectic induction. It turns out to be equivalent to the condition that the orbit in question is a symplectic subbundle of a modified cotangent bundle.

1. Introduction

One of the original goals of geometric quantization was to obtain a general method of constructing (irreducible) representations of Lie groups out of their coadjoint orbits. The idea was to generalize the Borel-Weil-Bott theorem for compact groups and Kirillov's results for nilpotent groups. Since then geometric quantization has led a somewhat dual life. On the one hand, in representation theory where it is called the orbit method (see [8] for a relatively recent review). On the other hand, in physics where it serves as a procedure that starts with a symplectic manifold (a classical theory) and creates a Hilbert space and a representation of the Poisson algebra as operators on it (the quantum theory).

Recent results in quantum reduction theory [5] allow us to show rigorously in some particular cases that geometric quantization intertwines the procedures of symplectic induction and unitary induction. Since the latter is one of the ingredients in the orbit method, this gives a geometrical insight into the "classical" part of the orbit method. In particular, it allows us to give a geometrical interpretation of Pukanszky's condition on a polarization which is completely different from the well-known interpretation that says that the coadjoint orbit contains an affine plane. In fact, we prove (Proposition 3.9) that Pukanszky's condition is equivalent to the statement that the coadjoint orbit in question is, in a noncanonical way, symplectomorphic to a symplectic subbundle of a modified cotangent bundle (where modified means that the canonical symplectic form on

Received June 6, 1990 and, in revised form, July 8, 1991.