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ON THE SPECTRAL GAP FOR COMPACT MANIFOLDS

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1. Introduction

We aim to give lower bounds for the spectral gap of the Laplace operator on a compact Riemannian manifold in terms of a lower bound for the Ricci curvature and an upper bound for the diameter of the manifold.

We apply the maximum principle technique to $|\nabla\varphi|^2 - G(\varphi)$ for appropriate auxiliary functions G. The auxiliary functions are chosen in such a way that the above quantity vanishes identically if φ is replaced by an eigenfunction of an appropriate Neumann boundary problem. For the case of manifolds with nonnegative Ricci curvature it is sufficient to consider radial eigenfunctions for annular regions in constant curvature spaces.

Our approach seems to yield better results than techniques using isoperimetric inequalities (cf. [1]). If additional information about the median value of an eigenfunction is known, a sharper estimate can be obtained which, in particular, improves the result by Zhong and Yang (see [11] and [6], §4]). Our basic examples show that the estimates are in some sense sharp.

2. Statement of the basic gradient estimate

We obtain our basic estimate by comparison with a Neumann problem on a manifold with boundary. The manifold is constructed using Fermi coordinates on a sphere of constant curvature with sufficiently small diameter. This construction is also closely related to the proof of the Lévy-Gromov isoperimetric inequality (see [3], §§XXII.8 and XII.9]). We adopt the notation of Chavel's book.

Let a dimension $n > 1$, a diameter d, and a constant Ricci curvature R be given. We set $\kappa \equiv R/(n-1)$. Suppose that the condition $d \leq \pi/\sqrt{\kappa}$

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