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L_2-COHOMOLOGY OF KÄHLER VARIETIES WITH ISOLATED SINGULARITIES

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0. Introduction

Let V be a complex projective variety. If V is smooth, we may apply the de Rham-Hodge theory to represent the cohomology by harmonic forms. As a consequence of this and the Kähler identities, we obtain a Hodge decomposition (or *Hodge structure*) on the cohomology,

$$H^k(V;\mathbb{C}) = \bigoplus_{p+q=k} H^{p,q}, \qquad H^{p,q} = \overline{H^{q,p}}.$$

When V is singular, such a decomposition no longer exists in general. However, by imposing restrictions on the intersections of chains with singular strata, Goresky and MacPherson [21], [22] defined an alternate cohomology theory for singular spaces. This *intersection cohomology*, $IH^{\cdot}(V;\mathbb{C})$, has many of the properties of ordinary cohomology on manifolds. For example, there is a nondegenerate Poincaré duality pairing, and by using the theory of $\mathscr{D}$-modules, Saito [38]–[40] (see also [41]) has shown that $IH^{\cdot}(V;\mathbb{C})$ admits a natural Hodge decomposition. (We are always referring to the *middle perversity* intersection cohomology. For an excellent historical introduction to intersection homology theory and its many ramifications, see Kleiman's article [28]; for an overview of recent advances in Hodge theory, see [10].)

It is natural to try to represent intersection cohomology analytically by a de Rham-type theory, and obtain another, more classical, proof of the existence of a Hodge decomposition on $IH^{\cdot}(V;\mathbb{C})$. In a number of contexts [11], [52] it has been conjectured that the appropriate de Rham theory to consider is the L_2-cohomology of some metric on $V\setminus \operatorname{Sing}(V)$, the set of regular points of V. Given a Riemannian metric on $V\setminus\operatorname{Sing}(V)$, the *$L_2$-cohomology* $H^{\cdot}_{(2)}(V\setminus\operatorname{Sing}(V))$ is the cohomology of the complex of L_2 differential forms whose exterior derivatives are also L_2. L_2-cohomology

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