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SHADOW LINKS AND FACE MODELS OF STATISTICAL MECHANICS

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Abstract

We introduce a new geometric technique enabling one to present links in oriented 3-manifolds, which are S^1-fibrations over oriented surfaces, by configurations of loops on the surfaces equipped with some additional data. This technique naturally leads to a purely 2-dimensional notion of shadow links on surfaces. We use IRF-models, based on the quantum $6j$-symbols associated with the Lie algebra $Sl_2(\mathbb{C})$, to construct invariants of shadow links generalizing the Jones polynomial of links in the 3-sphere S^3.

Introduction

Since the appearance of the Jones polynomial for links in the 3-sphere S^3, considerable efforts have been spent to construct analogous invariants for links in other 3-manifolds. An important breakthrough was made by E. Witten [16] who defined (on the physical level of rigour) Jones-type invariants of links in arbitrary closed 3-manifolds. Witten's approach is based on quantum field theory with a nonabelian Chern-Simons action.

A mathematical construction of "quantum" invariants of links generalizing the Jones polynomial was given in [10]. This construction is based on the representation theory of quantum groups and the surgery theory of manifolds. The surgery theory, developed in dimension 3 by V. Rochlin, W. Lickorish, R. Kirby, R. Fenn, and C. Rourke, enables one to reduce the study of links in closed 3-manifolds to the case of links in S^3 where the technique of R-matrices and categories of tangles is applicable (see [2], [9]–[14]). The invariants of links in 3-manifolds obtained in this way satisfy the same formal properties as the Witten invariants and therefore may be viewed as a mathematical realization of Witten's program.

Another approach to quantum invariants of compact 3-manifolds and links in these manifolds has been presented in [15]. The construction in [15] is based on the theory of quantum $6j$-symbols developed in [6] and

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