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RICCI FLOW OF LOCALLY HOMOGENEOUS GEOMETRIES ON CLOSED MANIFOLDS

JAMES ISENBERG & MARTIN JACKSON

Abstract

Hamilton's program for using Ricci flow to study Thurston's three-dimensional geometrization conjecture requires one to understand the Ricci flow of all locally homogeneous geometries on closed three-manifolds. We study these flows and describe their characteristic behaviors

1. Introduction

Thurston's three-dimensional geometrization conjecture [9], [7], claims that any closed three-manifold M^3 may be canonically decomposed into pieces such that each of the pieces admits a locally homogeneous geometry. Hamilton has proposed a program for proving this conjecture using Ricci flow. Roughly, the idea is to choose an arbitrary metric on M^3 and then deform this metric via the (normalized) Ricci flow equation

$$\frac{\partial}{\partial t}g = -2\mathrm{Ric} + \frac{2}{3}\langle R\rangle g, \tag{1}$$

where $\langle R\rangle$ denotes the average of the scalar curvature R over M^3. One hopes to relate the local singularities of the flow to the manifold decomposition in Thurston's conjecture, and then one hopes to show that the Ricci flow of the geometry away from each of the local singularities approaches that of a locally homogeneous geometry in each disconnected piece.

While it has been shown that the Ricci flow for certain classes of three-metrics converges ([3], [2]), there are many examples known of three-metrics whose Ricci flows do not converge. It is not surprising that three-dimensional Ricci flows do not generally converge. Ricci flows can only converge to Einstein metrics (the zeros of the right-hand side of the Ricci flow equation (1)), and most three-manifolds (e.g., $S^2 \times S^1$) do not admit an Einstein metric.

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