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A LIPSCHITZ DECOMPOSITION OF MINIMAL SURFACES

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1. Introduction

Let Γ be a simple closed rectifiable curve in Euclidean space $\mathbf{R}^n$. We say that Γ is an M *chord-arc curve* if $l(z, w) \le M|z-w|$ for all $z, w \in \Gamma$, where $l(z, w)$ denotes the length of the shorter subarc of Γ joining z to w. Let $\psi(e^{it})$, $0 \le t \le 2\pi$, parametrize such a curve Γ with $|\psi'(e^{it})| \equiv l(\Gamma)/2\pi$, where $l(\Gamma)$ denotes the length of Γ. Then for $0 \le t-s \le \pi$, we have

$$c_1 \le \frac{|\psi(e^{it}) - \psi(e^{is})|}{|e^{it} - e^{is}|} \le c_2 \tag{1.1}$$

with $c_2/c_1 \le \frac{\pi}{2}M$. In other words, Γ is a bi-Lipschitz image of the unit circle. Conversely, if (1.1) holds for some parametrization of Γ, then

$$l(\psi(e^{it}), \psi(e^{is})) \le \left(\frac{\pi}{2}\right)^2 M|\psi(e^{it}) - \psi(e^{is})| \tag{1.2}$$

and thus Γ is a $(\frac{\pi}{2})^2 M$ chord-arc curve.

By a *minimal surface with boundary* Γ we mean the image $F(\mathbb{D})$ of the open unit disk $\mathbb{D} = \{z \in \mathbb{C}: |z| < 1\}$ under a continuous map

$$F = (F_1, \cdots, F_n): \overline{\mathbb{D}} \to \mathbb{R}^n$$

from the closed disk to $\mathbb{R}^n$ such that

(1.3) $F|_{\partial\mathbb{D}}$ is a homeomorphism of ∂D onto Γ,

(1.4) $F|_{\mathbb{D}}$ is C^2,

(1.5) $f_j \equiv \dfrac{\partial F_j}{\partial x} - i\dfrac{\partial F_j}{\partial y}$, $\quad 1 \le j \le n$, $z = x + iy$, is analytic in $\mathbb{D}$,

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