J. DIFFERENTIAL GEOMETRY
35 (1992) 643-657

CONSTANT MEAN CURVATURE SURFACE, HARMONIC MAPS, AND UNIVERSAL TEICHMÜLLER SPACE

TOM Y. H. WAN

1. Introduction

It has been known for some years that the Gauss map of a space-like surface in Minkowski 3-space $\mathbf{M}^{2,1}$ is harmonic if and only if the mean curvature of the surface is constant (see [7]). Using this fact, we can construct, for each holomorphic quadratic differential on a simply-connected domain Ω in $\mathbf{C}$, an injective harmonic map from Ω into the Poincaré disk $\mathbf{D}$. This harmonic map is unique up to equivalent classes, provided that it satisfies a completeness condition. In the process, we will find a classification of all complete hyperbolic space-like surfaces of constant mean curvature in $\mathbf{M}^{2,1}$.

It is also known that harmonic maps between surfaces are deeply related to the Teichmüller theory of compact Riemann surfaces (see [9]). So, it is interesting to study the analogy for universal Teichmüller space. For this problem, we find that a harmonic diffeomorphism from $\mathbf{D}$ onto itself is quasi-conformal if and only if the associated Hopf differential is bounded with respect to the Poincaré metric. From this result, we have a continuous map from the space of equivalence classes of holomorphic quadratic differentials under the action of the Möbius group, which is bounded with respect to the Poincaré metric, into the universal Teichmüller space.

The author would like to thank Professor R. Schoen for his continuous encouragement and support during the last few years. Without his support, this paper would not have been possible.

2. Geometric preliminaries

The Minkowski 3-space $\mathbf{M}^{2,1}$ is $\mathbf{R}^2 \times \mathbf{R}^1$ endowed with the metric

$$ds^2 = (dx^1)^2 + (dx^2)^2 - (dx^3)^2,$$

Received January 15, 1991.