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GRAUERT TUBES AND THE HOMOGENEOUS MONGE-AMPÉRE EQUATION. II

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1. Introduction

Let M be a complex n-dimensional manifold and $\sigma\colon M \to M$ an anti-holomorphic involution. The fixed point set X of σ is an n-dimensional real-analytic submanifold of M which is "totally real" at all points p (i.e., there exists no nonzero holomorphic vector in $T_pM \otimes \mathbf{C}$ with the property that both its real and its imaginary part are tangent to X). To simplify the exposition below we will also assume that X is compact (though a good deal of what we have to say in the following paragraph is true without this assumption). We recall that the article [8], of which this article is a continuation, has to do with the following well-known theorem of Grauert:

Theorem. *There exists a σ-invariant neighborhood M_1 of X in M and a smooth strictly plurisubharmonic function $\rho\colon M_1 \to [0, 1)$, such that*

$$X = \rho^{-1}(0) \quad \textit{and} \quad \sigma^*\rho = \rho. \tag{1.1}$$

The main result of [8] is that the function, ρ, in this theorem can be chosen to have an additional property: namely to satisfy the homogeneous Monge-Ampére equation

$$\det\left(\frac{\partial}{\partial z_i \partial \overline{z}_j}\sqrt{\rho}\right) = 0 \tag{1.2}$$

on the compliment of X in M_1. In fact we showed that if X is equipped with a real-analytic Riemannian metric, there exists a unique real analytic solution ρ of (1.2) such that the inclusion of X into M_1 is an isometric imbedding of X (equipped with the given metric) into M_1 equipped with the Kaehler metric

$$\frac{1}{\sqrt{-1}}\sum \frac{\partial^2 \rho}{\partial z_i \partial \overline{z}_j}\, dz_1\, d\overline{z}_j. \tag{*}$$

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