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COMPACTNESS THEOREMS FOR KÄHLER-EINSTEIN MANIFOLDS OF DIMENSION 3 AND UP

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There has been increasing interest lately in compactness theorems of Riemannian manifolds under various geometric assumptions (see, among others, [3], [10], [1], [7], and [19]). More recently, it has been found that the boundedness condition on the curvature as in [3] and [10] can be replaced by some integral norms of the curvature tensor. One of those often used is the $L^{n/2}$-norm on the curvature tensor, where n is the real dimension of the underlying manifold. For instance, in [1] and [19], the authors show that if $\{(M_i, g_i)\}$ is a sequence of Einstein manifolds of real dimension $2n$ satisfying: (i) $\mathrm{diam}(M_i, g_i) \leq \mu$; (ii) $\int_{M_i} \|Rm(g_i)\|^n_{g_i} dV_{g_i} \leq \mu$; and (iii) $\mathrm{Vol}(M_i, g_i) \geq \frac{1}{\mu}$, where μ is a uniform constant, then the subsequence of $\{(M_i, g_i)\}$ converges to an Einstein orbifold with finitely many isolated singular points. Also see [20] for the case of Kähler-Einstein surfaces. The case that the limit is an orbifold does occur in dimension four (cf. [15], [20]). However, in this paper, we show that it cannot occur for Kähler-Einstein manifolds of higher dimension and nonzero scalar curvature. In order to give our main theorem precisely, we need to introduce some notation first. For any fixed constant $\mu > 0$ and positive integer $n > 0$, denote by $K(\mu, n)$ the set of all Kähler-Einstein manifolds (M, g) of complex dimension n satisfying:

$$\mathrm{diam}(M, g) \leq \mu, \tag{0.1}$$

$$\int_M |Rm(g)|^n_g \, dV_g \leq \mu, \tag{0.2}$$

$$\mathrm{Vol}_g(M) \geq 1/\mu, \tag{0.3}$$

where $Rm(g)$ denotes the curvature tensor of g. Let $K_+(\mu, n)$ (resp. $K_-(\mu, n)$) be the subset of all (M, g) in $K(\mu, n)$ with $\mathrm{Ric}(g) = \omega_g$ (resp. $\mathrm{Ric}(g) = -\omega_g$), where ω_g is the associated Kähler form of g. We should point out that the diameters of the manifolds in $K_+(\mu, n)$ are

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