J. DIFFERENTIAL GEOMETRY
35 (1992) 521-534

POLYCYCLIC GROUPS AND TRANSVERSELY AFFINE FOLIATIONS

J. F. PLANTE

Introduction

A foliation is called transversely affine if coordinates can be chosen so that the holonomy maps are all affine [3]. It is known that there is a close relation between complete affinely flat manifolds and polycyclic groups. Specifically, all polycyclic groups occur as the fundamental group of such a manifold [10] and it is conjectured that the converse is virtually true. Here we consider foliations of codimension one of manifolds with polycyclic fundamental group and show that in certain natural situations the foliation must have a transverse affine structure. For example, when the manifold is compact with polycyclic fundamental group, any real analytic foliation with exponential growth (of all leaves) is transversely affine (Theorem 4.1). For foliations with less differentiability analogous results are obtained by making additional topological hypotheses on the foliation. These results may be thought of as generalizations of results from [4] and [17] for manifolds of dimension three. In contrast, examples from [8] are described which show that these results do not hold when the fundamental group is merely assumed solvable. The results for foliations are based on a study of smooth actions by polycyclic groups on the real line which, using the main result in [23], yields sharper conclusions than similar results obtained in [18] for actions by more general solvable groups.

1. Polycyclic groups of diffeomorphisms of $\mathbb{R}$

Denote by $\mathrm{Diff}^k(\mathbb{R})$ the group of $\mathscr{C}^k$ diffeomorphisms of the real line, where k is a positive integer, ∞, or ω (real analytic). We will denote by $\mathrm{Aff}(\mathbb{R})$ the subgroup of $\mathrm{Diff}^\omega(\mathbb{R})$ consisting of affine maps ($x \mapsto ax+b$ for some $a \neq 0$, $b \in \mathbb{R}$). Of particular interest to us will be diffeomorphism groups which are polycyclic and have exponential growth. For abstract groups of this type the basis reference is Wolf [22].

Received July 25, 1990.