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HARMONIC FUNCTIONS AND THE STRUCTURE OF COMPLETE MANIFOLDS

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Dedicated to Professor Shiing-Shen Chern on his 79th birthday

0. Introduction

This paper is motivated by previous work of the authors [18] and its application to the study of the structure of complete Kähler manifolds in a subsequent work of the first author [16]. Roughly speaking the main theorem in [18] relates the infinity geometric structure of a certain class of manifolds to the theory of harmonic functions. Let us recall the precise setting.

Let M be a complete noncompact manifold without boundary. Suppose the sectional curvature K_M of M is nonnegative outside some compact subset of M. Without loss of generality, we may assume that the compact subset is contained in a geodesic ball of radius 1. By using the argument of Cheeger-Gromoll [5], one concludes rather easily that M has finite topological type. In particular, M has finitely many ends and each end is homeomorphic to the product of a compact manifold with the half-line. In fact, Abresch [1] (also see [2]) proved that a slightly more general assumption on the sectional curvature, which he referred to as asymptotically nonnegatively curved, is sufficient to imply that M has finite topological type. Moreover, the number of ends of M can be estimated by a quantity computable by the pointwise lower bound of the sectional curvature.

In the case when M has nonnegative sectional curvature on $M\backslash B(1)$, the ends of M are given by the connected components of $M\backslash B(1)$. The notions of large and small ends were defined by the authors [18] depending on the volume growth of the end. With these definitions, we may assume the ends of M are given by s small ends $\{e_1, \cdots, e_s\}$ and l large ends $\{E_1, \cdots, E_l\}$, for some $0 \leq s < \infty$ and $0 \leq l < \infty$ with $s + l \geq 1$. The main result of [18] was to show that the numbers s and l have

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