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A KNOT INVARIANT VIA REPRESENTATION SPACES

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0. Introduction

A beautiful construction of A. Casson on the representation spaces corresponding to a Heegard splitting of an oriented homology 3-sphere M gives rise to an integer invariant $\lambda(M)$ of M. This invariant generalizes the Rohlin invariant and gives stricking corollaries in low-dimensional topology. Defined as an intersection number of appropriate subspaces, this invariant $\lambda(M)$ can be roughly thought of as the number of conjugacy classes of irreducible representations of $\pi_1(M)$ into SU_2 counted with signs. A detailed discussion of this invariant can be found in an exposé by S. Akbulut and J. McCarthy [1]. Further works on Casson's invariant include the generalizations by K. Walker as well as S. Boyer and A. Nicas to rational homology 3-spheres [14], [3] and by S. Cappell, R. Lee, and E. Miller to representations into SU_n [4]. The works of C. Taubes [13] and A. Flore [6] interpret Casson's invariant as the Euler number of the instanton homology (or Flore homology) of M.

In this paper, analogous to Casson's original construction, we will define an intersection number of the representation spaces corresponding to a braid representative of a knot K in S^3. This intersection number turns out to be an integer knot invariant (see Theorem 1.8). The representations of the knot group $\pi_1(S^3\backslash K)$ used in our construction seem to be mysterious. They are representations of $\pi_1(S^3\backslash K)$ into SU_2 such that all meridians of K are represented by trace-zero matrices. Call such a representation of the knot group a *trace-free representation.* Then, roughly speaking, our knot invariant $h(K)$ is the number of conjugacy classes of irreducible trace-free representations of $\pi_1(S^3\backslash K)$ counted with signs.

Our knot invariant $h(K)$ can be computed via knot diagrams. It turns out that our algorithm of computing $h(K)$ by using the skein model is the same as the algorithm of computing $\frac{1}{2}\mathrm{sgn}(K)$ given by J. H. Conway [5],

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