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C^α-COMPACTNESS FOR MANIFOLDS WITH RICCI CURVATURE AND INJECTIVITY RADIUS BOUNDED BELOW

MICHAEL T. ANDERSON & JEFF CHEEGER

0. Introduction

In this note, we consider the class of Riemannian n-manifolds (M, g) which have a lower bound on the Ricci curvature and on the injectivity radius

$$\mathrm{Ric}_M \geq -\lambda, \qquad \mathrm{inj}_M \geq i_0. \tag{0.1}$$

Our main result is that the C^α geometry of the metric g and the $C^{1,\alpha}$ topology of the manifold M are controlled by these bounds. More precisely, we obtain

Theorem 0.1. *Let (M, g) be a compact Riemannian manifold satisfying the bounds*

$$\mathrm{Ric}_M \geq -\lambda, \qquad \mathrm{inj}_M \geq i_0, \quad \mathrm{vol}_M \leq V. \tag{0.2}$$

Then for all $\alpha < 1$ and $Q > 1$, there is a finite atlas of harmonic coordinate charts $F_\nu \colon U_\nu \to \mathbb{R}^n$ for M, having the following properties:

(1) *The domains U_ν are of the form $U_\nu = F_\nu^{-1}(B(r_h))$, $B(r_h)$ a ball in $\mathbb{R}^n$, of radius r_h, satisfying*

$$r_h \geq C(\lambda, i_0, n, \alpha, Q).$$

Further, the domains $F_\nu^{-1}(B(r_h/2))$ cover M.

(2) *The overlaps $F_{\mu\nu} = F_\mu \circ F_\nu^{-1}$ are controlled in the $C^{1,\alpha}$ topology, i.e.,*

$$\|F_{\mu\nu}\|_{C^{1,\alpha}} \leq C(\lambda, i_0, n, \alpha, Q).$$

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