J. DIFFERENTIAL GEOMETRY
35 (1992) 219-254

UNCOUNTABLY MANY EXOTIC $\mathbf{R}^4$'S IN STANDARD 4-SPACE

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Abstract

It is known that the standard (Euclidean) smooth structure on 4-space when restricted to certain open subsets homeomorphic to $\mathbf{R}^4$ gives a smooth structure which is not diffeomorphic to the standard one. This behavior is a consequence of Donaldson's counterexample [5] to the smooth 5-dimensional h-cobordism theorem and was noticed (in anticipation of Donaldson's result) by A. Casson and the second named author (see [14, Theorem 3, Chapter 14]). Taubes [24] developed a technically demanding theory of the Yang-Mills equation on "asymptotically end periodic" 4-manifolds in part to verify that a known family of exotic $\mathbf{R}^4$'s were mutually distinct. That family lays smoothly in $S^2 \times S^2$ but not $\mathbf{R}^4$. We combine ideas from the above-mentioned papers to address a nested family of $\mathbf{R}^4$ homeomorphs called "ribbon $\mathbf{R}^4$'s" lying in $\mathbf{R}^4$ standard. There are continuum many pairwise distinct smooth structures represented within this family.

0. Introduction

Our philosophy is that any Donaldson-style invariant [5] can be defined on an "end periodic" manifold and these invariants commute with the passage between a compact manifold and such noncompact geometric limits. In principle the Γ-invariant or "polynomial-invariant" is suitable for this discussion; however, we carry out the analysis in detail only for D. Kotschick's "simpler" Φ-invariant [16]. Kotschick distinguishes a certain algebraic surface, the Barlow surface B, from the rational surface $Q = CP^2 \# 8\overline{CP}^2$ by showing that $|\Phi(B)| \geq 4$ and $\Phi(Q) = 0$. Taubes paper [24] on the self-dual Yang-Mills equation on end periodic 4-manifolds provides much of the technical foundation for our extension.

It is known that B and Q are smoothly h-cobordant (and therefore homeomorphic); that is, there exists $(W^5; B, Q)$ with $\partial W^5 = B \amalg -Q$, and the inclusions $B \hookrightarrow W^5$, $Q \hookrightarrow W^5$ are homotopy equivalences. It is, by now, a standard idea that W^5 should be analyzed with a mind toward

Received June 11, 1990 and, in revised form, March 20, 1991. The authors were supported in part by National Science Foundation Grant DMS-89-01412.