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CONFORMAL FIELD THEORY AND THE COHOMOLOGY OF THE MODULI SPACE OF STABLE BUNDLES

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1. Introduction

Let Σ_g be a compact Riemann surface of genus $g \geq 2$, and let Λ be a line bundle over Σ_g of degree 1. Then the moduli space of rank-2 stable bundles V over Σ_g such that $\Lambda^2 V \cong \Lambda$ was shown by Seshadri [17] to be a nonsingular projective variety $\mathcal{N}_g$. Its rational cohomology ring $H^*(\mathcal{N}_g)$ is exceedingly rich and despite years of study has never quite been computed in full. The object of this paper is to give an essentially complete characterization of this ring, or at least to reduce the problem to a matter of linear algebra. In particular, we find a proof of Newstead's conjecture that $p_1^g(\mathcal{N}_g) = 0$. We also obtain a formula for the volume of $\mathcal{N}_g$, which can be regarded as a twisted version of the formula for the degree 0 moduli space recently announced by Witten.

The approach which we shall take is not from algebraic geometry but from mathematical physics: it relies on the SU(2) Wess-Zumino-Witten model, which is a functor Z_k associating a finite-dimensional vector space to each Riemann surface with marked points. The relationship with the moduli space is that when the "level" k of the functor is even, the vector space associated to Σ_g with no marked points can be identified with $H^0(\mathcal{N}_g; L^{k/2})$, where L is a fixed line bundle over $\mathcal{N}_g$. Now the work of Verlinde provides us with a means of calculating the dimension of any vector space arising from our functor, and in particular $\dim H^0(\mathcal{N}_g; L^{k/2})$, which we shall denote $D(g, k)$. On the other hand Newstead [13] found explicit generators for $H^*(\mathcal{N}_g)$, and we can also express $D(g, k)$ in terms of them using a Riemann-Roch theorem. Equating the two formulas enables us to evaluate any monomial in the generators on the fundamental class of $\mathcal{N}_g$, and by Poincaré duality this is sufficient, at least in principle, to determine the ring structure of $H^*(\mathcal{N}_g)$.

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