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ALMOST CONVEX GROUPS, LIPSCHITZ COMBING, AND π_1^∞ FOR UNIVERSAL COVERING SPACES OF CLOSED 3-MANIFOLDS

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Abstract

If $\pi_1 M^3$ is almost convex, then $\pi_1^\infty \widetilde{M}^3 = 0$. Under a mild restriction, the same conclusion holds if $\pi_1 M^3$ admits a Lipschitz combing in the sense of Thurston.

1. Introduction

The main result of this paper is that if M^3 is a closed 3-manifold such that $\pi_1 M^3$ is almost convex, then the universal covering space $\widetilde{M}^3$ is simply-connected at infinity. We start by recalling what "almost convex" means.

We consider a finitely generated group G and a specific finite set of generators $B = B^{-1}$ for G. To this, we can attach the Cayley graph $\Gamma = \Gamma(G, B)$. For each $g \in G$, we will denote by $\|g\|$ the minimal length of a word with letters in B expressing g. We also define $d(g, h) = \|g^{-1}h\| = \|h^{-1}g\|$.

For any positive integer, we can consider the ball of radius n in Γ,

$$B(n) \underset{\text{def}}{=} \{x \in \Gamma \text{ such that } \|x\| \le n\}, \tag{1.1}$$

and the sphere of radius n in Γ,

$$S(n) \underset{\text{def}}{=} \{x \in \Gamma \text{ such that } \|x\| = n\}. \tag{1.2}$$

Following J. Cannon [3], we will say that the Cayley graph $\Gamma = \Gamma(G, B)$ is *k-almost convex* (for $k \in Z^+$) if there exists an $N = N(k) \in Z^+$ with the property that for any n if $x, y \in S(n)$ are such that $d(x, y) \le k$, then x and y can be joined in $B(n)$ by a path of length $\le N(k)$. If, for

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