J. DIFFERENTIAL GEOMETRY
34 (1991) 681-700

# NONNEGATIVELY CURVED LEAVES IN FOLIATIONS

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**Abstract**

We use techniques from geometric analysis to prove that any Riemannian foliated measure space with finite total measure and leaves of nonnegative Ricci curvature has the property that a.e. leaf is the product of a compact Riemannian manifold and a flat Euclidean space

## 0. Introduction

We solve here a conjecture of R. Zimmer concerning nonnegatively curved leaves in foliations by Riemannian manifolds. We prove

**Theorem 5.1.** *Let $(M, \mathcal{F}, \mathcal{R})$ be a Riemannian foliated measure space with finite total measure such that a.e. leaf is complete and has nonnegative Ricci curvature. Then a.e. leaf can be written as the product of a compact Riemannian manifold and a flat Euclidean space.*

Thus, while it is possible to foliate a torus by lines, this theorem implies, for example, paraboloids cannot wrap around one another tightly enough to foliate a space of finite volume.

Theorem 5.1 is the counterpart of Zimmer's theorem [12] on nonpositively curved leaves in amenable foliations. In nonnegative curvature, it is not necessary to assume amenability; this comes for free since the leaves of the foliation have polynomial growth.

Our result, proved in the general context of foliated measure spaces, immediately implies the following theorem for foliations of Riemannian manifolds with holonomy-invariant measure (as defined for example in [8]):

**Corollary 5.2.** *Let $\mathcal{F}$ be a foliation of a compact manifold with a holonomy-invariant measure that is finite on compact subsets of transversals. Assume that almost every leaf (with respect to this measure) is a complete Riemannian manifold of nonnegative Ricci curvature. Then almost every*

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Received September 18, 1990. The authors' research was partially supported by National Science Foundation Grant Nos. DMS 89-09876 and DMS 88-13977, respectively.