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# A NEW SUPER KP SYSTEM AND A CHARACTERIZATION OF THE JACOBIANS OF ARBITRARY ALGEBRAIC SUPER CURVES

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**Abstract**

A set of super-commuting vector fields is defined on the super Grassmannians. A characterization of the Jacobian varieties of super curves (super Schottky problem) is established in the following manner: Every finite-dimensional integral manifold of these vector fields has a canonical structure of the Jacobian variety of an algebraic super curve, and conversely, the Jacobian variety of an arbitrary algebraic super curve is obtained in this way. The vector fields restricted on the super Grassmannian of *index* 0|0 give a completely integrable system of partial super differential equations which gives a new supersymmetric generalization of the KP system. Thus every finite-dimensional solution of this new system gives rise to a Jacobian variety of an algebraic super curve. The correspondence between this super Grassmannian and the group of monic super pseudodifferential operators of order zero (the super Sato correspondence) is also established.

## 0. Introduction

The purpose of this paper is to establish a characterization of the Jacobian varieties of arbitrary algebraic super curves defined over a field $k$ of characteristic zero by using certain super-commuting vector fields on the super Grassmannians.

Since our main theorem is a supersymmetric generalization of the characterization theorem of usual Jacobian varieties obtained in [5], let us sketch the nonsupersymmetric situation first. Consider the ring $k[[x]]$ of formal power series in one variable $x$ with coefficients in the field $k$, and a formal pseudodifferential operator

$$S = 1 + s_1(x)\left(\frac{\partial}{\partial x}\right)^{-1} + s_2(x)\left(\frac{\partial}{\partial x}\right)^{-2} + \cdots \tag{0.1}$$

of order zero with coefficients in $k[[x]]$. The Kadomtsev-Petviashvili

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