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GEOMETRIC CONSTRUCTION OF HOLONOMY COVERINGS FOR ALMOST FLAT MANIFOLDS

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1. Introduction

In this paper we give a new and conceptually rather simple proof of Gromov's theorem on almost flat Riemannian manifolds ([7], [2], [12], [3]). The proof yields a generalized version of the result which can be stated as follows.

1.1. Theorem. *There is a positive constant* $\varepsilon(n)$ *depending only on* n *such that the following is true. Let* (M, g) *be a compact connected* n*-dimensional Riemannian manifold,* d *its diameter, and* ∇ *a connection on* TM *compatible with the metric* g. *If the curvature and torsion tensors* R *and* T *of* ∇ *satisfy*

$$(1.1.1)\qquad (\|R\|_\infty + \|T\|_\infty^2)\, d^2 \le \varepsilon(n),$$

then M *is diffeomorphic to an infranilmanifold* $N = \Lambda \backslash G$.

Here $\|\cdot\|_\infty$ denotes the maximum norm on tensors. Infranilmanifolds are defined in §1.2. The constant $\varepsilon(n)$ is effective, but no explicit bound will be given. The nilpotent group structure on G is determined by the fundamental group Λ of M (see [1]).

The case $T = 0$ is Gromov's theorem as sharpened by Ruh [12]. The case $R = 0$ yields a generalization of [5]. Finally, the locally homogeneous case $R = 0$ and $\nabla T = 0$ is essentially due to Kazhdan-Margulis (see [11], [4]).

Previous proofs required a detailed study of what is known as the local fundamental pseudogroup of M (see [7], [2], [3]). The present proof uses results obtained in [5], [6], and is based on a description of the structure of distance balls in the bundle P of orthonormal frames on M, making essential use of the fact that P is parallelizable with torsion bounded in terms of the Cartan curvature. The holonomy covering space $\Gamma \backslash G$ (see §1.2) associated with a flat connection with parallel torsion on M is

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