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GRAUERT TUBES AND THE HOMOGENOUS MONGE-AMPÈRE EQUATION

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1. Introduction

Let X be a compact real-analytic manifold of dimension n. A theorem of Bruhat and Whitney [2] states that there exists a complex "thickening" of X: a complex n-dimensional manifold, M, and a real-analytic imbedding of X in M with the property that, as a submanifold of M, X is totally real. (Meaning that if $p \in X$, and J_p is the defining map for the complex structure on T_pM, then $J_p(T_pX)$ intersects T_pX in $\{0\}$.) In addition one can arrange that there exists on M an antiholomorphic involution

$$\sigma: M \to M \tag{1.1}$$

whose fixed point set is X. Suppose now that X is equipped with a Riemannian metric. What we will be concerned with in this paper is the following question: Can one find a Kaehler structure on M which is in some way "intrinsically associated" with the Riemannian structure on X? Before posing this question in a more precise form we will first say a few words about Grauert tubes; In [4] Grauert showed that there exist a neighborhood M_1 of X in M and a smooth strictly plurisubharmonic function

$$\rho: M_1 \to [0, 1) \tag{1.2}$$

with $X = \rho^{-1}(0)$ and $\rho(\sigma(m)) = \rho(m)$ for all $m \in M_1$. The fact that ρ is strictly plurisubharmonic implies that the open sets

$$M_\varepsilon = \rho 6-1([0, \varepsilon)), \qquad 0 < \varepsilon < 1, \tag{1.3}$$

are strictly pseudoconvex and hence possess lots of globally defined holomorphic functions. From this Grauert was able to deduce that X itself possesses a lot of globally defined real-analytic functions. (The fact that a real-analytic manifold possesses a lot of globally defined real-analytic

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