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SINGULARITIES OF THE CURVE SHRINKING FLOW FOR SPACE CURVES

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Abstract

Singularities for space curves evolving by the curve shrinking flow are studied. Asymptotic descriptions of regions of the curve where the curvature is comparable to the maximum of the curvature are given.

PART I. OVERVIEW

1. Introduction. In this work, we will study singularity formation for space curves evolving by the curve shortening flow

$$\frac{\partial \gamma}{\partial t} = k \cdot N, \tag{1.1}$$

where $\gamma : S^1 \times [0, \omega) \to R^3$, $\gamma(\cdot, 0)$ is a smooth curve, and $k \cdot N$ is the curvature times the normal to the curve. N is not always defined, though $k \cdot N$ always makes sense.

Although one has short time existence of solutions on a small open interval in time [7], solutions do not exist for infinite time. In a previous paper [2], it has been shown that solutions to the space curve flow exist until the curvature becomes unbounded. In this work, we study the limiting shape of the curve along forming singularities.

Space curves, in general, behave in a more complicated manner than plane curves. For example, they may not remain embedded (see Figure 1) and inflection points may develop [2]. We prove the rather surprising conjecture, due to Matt Grayson, that singularity formation is a planar phenomenon. We then give asymptotic descriptions of the solution.

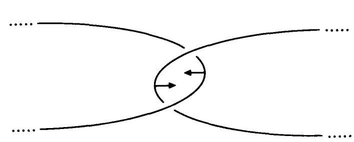

FIGURE 1. CURVES CAN CROSS

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