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ANALYTIC TORSION FOR GROUP ACTIONS

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I. Introduction

The Reidemeister torsion is a classical topological invariant for non-simply-connected manifolds [14]. Let M be a closed oriented smooth manifold with the fundamental group $\pi_1(M)$ nontrivial. Let $\rho: \pi_1(M) \rightarrow O(N)$ be an orthogonal representation of $\pi_1(M)$. If the twisted real cohomology $H^*(M, E_\rho)$ vanishes, then one can define the Reidemeister torsion $\tau_\rho \in \mathbb{R}$, which is a homeomorphism invariant of M. The original interest of τ_ρ was that it is not a homotopy invariant, and so can distinguish spaces which are homotopy equivalent but are not homeomorphic.

Ray and Singer asked whether, as for many other topological quantities, one can compute τ_ρ by analytic methods [17]. Given a Riemannian metric g on M, they defined an analytic torsion $T_\rho \in \mathbb{R}$ as a certain combination of the eigenvalues of the Laplacian acting on twisted differential forms. They showed that under the above acyclicity condition, T_ρ is independent of the metric g, and they conjectured that the analytic expression T_ρ equals the combinatorial expression τ_ρ. This conjecture was proven to be true independently by Cheeger [4] and Müller [15].

One can look at the above situation in the following way. The group $\pi_1(M)$ acts freely on the universal cover $\widetilde{M}$, and so one has an invariant for free group actions. A natural question is whether the Reidemeister torsion can be extended to an invariant for more general group actions. For a finite group acting (not necessarily freely) on a closed oriented PL manifold X, a Reidemeister torsion was defined algebraically by Rothenberg [19]. One can then ask whether there is a corresponding analytic torsion when X is smooth, and whether the analytic torsion equals the combinatorial torsion.

In §II we define the analytic torsion T_ρ for a finite group action and show that if the relevant cohomology groups vanish, then it is independent of the G-invariant metric used in its definition. (This was shown previously in unpublished work by Cheeger [5].) The analysis involved to

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