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REGULARITY FOR THE HARVEY-LAWSON SOLUTIONS TO THE COMPLEX PLATEAU PROBLEM

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1. Introduction

It seems that one of the natural fundamental questions of complex geometry is the classical complex Plateau problem. Specifically, the problem asks which odd-dimensional, real submanifolds of $\mathbf{C}^N$ are boundaries of complex submanifolds in $\mathbf{C}^N$.

Recall that a C^1-submanifold M of a complex manifold X is said to be maximally complex if

$$\operatorname{codim}_{\mathbf{R}}(T_x M \cap J(T_x M)) = 1 \quad \text{for all } x \in M,$$

where J denotes the almost complex structure of X, and the codimension refers to M. It was a fundamental contribution to complex geometry by Harvey and Lawson [3] that if M is compact, oriented, and of dimension larger than 1, and if X is Stein, then maximal complexity implies that M forms the boundary of a holomorphic n-chain in X.

If M is a CR-manifold in the sense of Kohn [6], [2] (see Definition 2.1 below), then there is a natural filtration associated to the De Rham complex of M with complex coefficients [8], [9]. The $E_1^{p,q}$ term of the spectral sequence associated to this filtration is called the Kohn-Rossi cohomology group $H_{\mathrm{KR}}^{p,q}(M)$ of M [7], [8], [9]. In [9], we gave smooth solutions to the classical complex Plateau problem in the following cases.

Theorem 1. *Let M be a compact, orientable, connected CR-manifold of real dimension $2n-1$, $n \geq 3$, in a Stein manifold X of complex dimension $n+1$. Suppose that M is strongly pseudoconvex. Then M is a boundary of a complex submanifold $V \subseteq X - M$ if and only if Kohn-Rossi's cohomology groups $H_{\mathrm{KR}}^{p,q}(M)$ are zero for $1 \leq q \leq n-2$.*

However, for strongly pseudoconvex (see Definition 2.4 below) CR-manifolds of real dimension three in $\mathbf{C}^3$, the smoothness of Harvey-Lawson solutions to the classical complex plateau problem remains open.

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